PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
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On the Richardson Gyro-Magnetic Effect

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VII. *On the Richardson Gyro-Magnetic Effect.*

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On the assumption that the molecular magnets in a ferro-magnetic substance consist of revolving electric charges associated with inertia, and that a change in any direction of the magnetisation of such a substance consequently implies the appearance in that direction of angular momentum which was not previously there, it is to be expected, as O. W. Richardson pointed out in 1908, that this change will also be accompanied by an angular impulse of reaction of which the magnitude is therefore equal and opposite to that of the corresponding angular momentum of the revolving charges. Such an impulse has now been observed by several experimenters. It is situated in the magnetic substance itself, and its sign points to the rotation of negative electricity as its cause.

Richardson showed further that if the revolving charges are of one sign only, m and e representing their masses and charges respectively, the ratio of the angular momentum to the corresponding change in magnetic moment (M) is given by the equation

$$\frac{\text{angular momentum}}{\text{M}} = \frac{2m}{e}.$$

For the particular case of revolving electrons $2m/e$ has the value $1 \cdot 13 \times 10^{-7}$.

Accurate experimental determination of this ratio is difficult. The values so far obtained, while differing among themselves, are, in general, lower than the theoretical ratio for electrons; the more dependable lying near to one-half of that quantity—so near indeed that the chief interest attaching to them now is in the question whether their value is exactly one-half or only approximately so.

The following is the account of an attempt to re-determine the ratio from this point of view.

The chief experimental difficulty lies, of course, in the fact that the reaction impulse tends to be masked, or at least seriously modified, by the disturbing forces consequent on the change of magnetism.

Three methods of obtaining its value appear to be available.

(*a*) A *ballistic method* (used by Stewart) in which the impulse turns a vertical cylinder against the twist of a delicate fibre by which it is suspended.

(*b*) A *resonance method* (used by EINSTEIN and DE HAAS and others) in which the angle of twist is increased by repeating the impulse at intervals of time equal to the period of the suspended cylinder.

(*c*) A *null method* (at which we are now working)* in which the twisting effect of the timed impulses is balanced by a series of equal and opposite impulses of known value applied simultaneously.

In spite of the fact that STEWART'S results with the ballistic method varied considerably among themselves a comparison of *a* with *b* (*c* had not occurred to us when we began the present work) seemed to show, not only that the sources of error would be more easily disentangled and allowed for in *a* than in *b*, but that *a* might be modified in the direction of greater accuracy than STEWART† obtained. We have therefore based the work recorded in what follows upon the ballistic method.

Apparatus.

As in STEWART'S measurements, the specimen to be tested was a piece of thin wire, in our case straightened by heating to redness under tension. This is shown at S, fig. 1, suspended vertically along the axis of the magnetising helix, H, by a fine quartz fibre attached to a torsion head K. K turns in a brass tube E, fixed at its lower end by the flange F to a horizontal wooden shelf, and held co-axial with the helix at its upper end by an annular brass ring L. For the sake of clearness the diameter of the helix is shown too large in proportion to the rest of the figure. Each end of the helix is adjustable laterally by four horizontal brass screws (not shown) which bear against the projecting ends of the brass tube on which the helix is wound. The quartz fibre is supported by a short brass rod which slides centrally through K, so that the height of the specimen in the helix may be adjusted.‡ A second brass rod, N, also slides through K excentrically, and carries two small curved pieces against which the specimen is made to rest by tilting when carrying it about for any purpose. This has proved a useful safeguard against the breaking of fibres by vibration. (A larger view of the torsion head and suspension is given at fig. 2A, p. 7.) The turning of the specimen on magnetisation was measured by a telescope and scale (T_1 S_1), the light reflected from the mirror attached to the specimen being brought to and from the latter by two 45 degree mirrors (M_1 M_2)

* In our arrangement, the balancing impulses are derived from currents induced by the changes of M in a helix surrounding the specimen ; these currents in turn acting on a small magnetic needle attached to the suspended system. In this way the method, while profiting by the enhanced sensitiveness due to resonance, does not involve the measurement of either frequency or damping, and possesses the further advantage that it may be made independent of time lag in the magnetic changes, and may therefore be used for the study of such substances as the Heusler alloys.

† His final value for iron is given to within ± 8 per cent., and for nickel to within ± 22 per cent.

‡ The specimen was always adjusted to hang with its centre a few millimetres below the centre of the helix to avoid any downward pull of the latter upon it which might break the fibre.

as in STEWART's apparatus. These mirrors were attached to the opposite ends of a brass tube made to slide easily in E; and, in addition to the vertical adjustment thus obtained, each mirror could be tilted.

STEWART measured the magnetic moment of his specimen by deflecting it slightly from the vertical with a known horizontal field, and we originally intended to use the same method, but difficulties of adjustment led us to replace it by the following magnetometer method. A, B, C, D (fig. 1) are four wooden shelves through each of which pass vertically three brass cylinders with screw threads cut upon them so that they may be

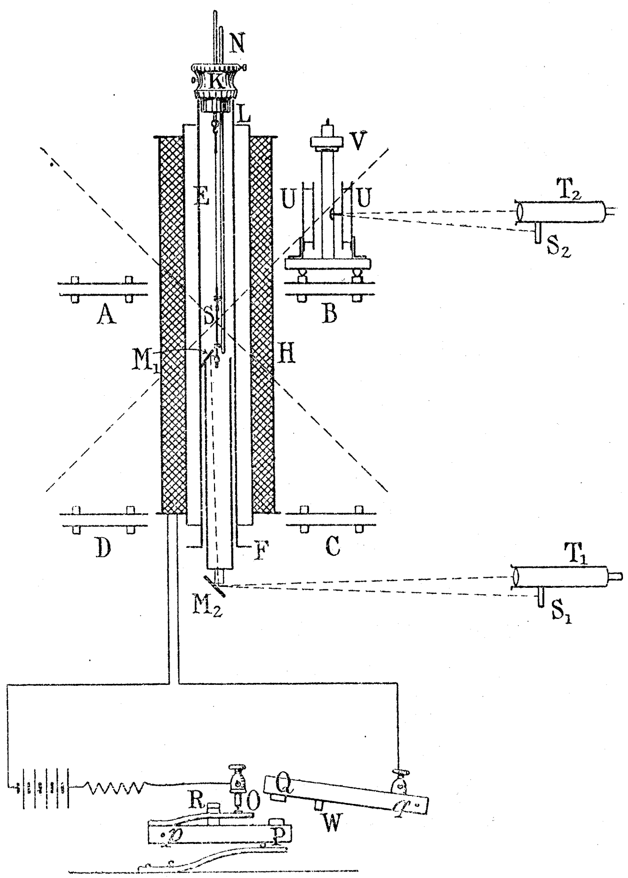


Fig. 1.

raised or lowered by turning in the wood. These cylinders meet three brass pieces projecting below the base of the magnetometer, and engage with them on the point, slot and plane principle. In this way the magnetometer could be rapidly set up in succession on A, B, C and D with the certainty that it had been moved parallel to itself if the cylinders had been properly adjusted.

The magnetometer, which was really a small tangent galvanometer with two coils U, U, and a small control magnet V, is shown standing on B. A, B, C, D are so placed that its needle, which hangs at the centre of the coils, lies upon one of the two dotted lines shown running through the centre of the specimen at 45 degrees to the vertical. The

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

four positions of the needle thus form the four corners of a square, of which the centre coincides with the centre of the specimen.

As the magnetic moment to be measured was small, it was necessary to place the needle near the specimen. The side of the square was therefore made 25·35 cm., and this necessitated correction for the distance, $2l$, between the poles. By direct measurement we found l to be 4 cm. to within a couple of millimetres—the error introduced in the effect of the specimen on the magnetometer by an error of 5 mm. in l being only 0·5 per cent.* It is therefore unlikely that there was any length error in our work greater than one- or two-tenths per cent.

The following is the expression for the horizontal component, H, of the field at the magnetometer needle due to the specimen

$$\mathrm{H} = \frac{\mathrm{M}d}{(d^2-\frac{1}{2}l^2)^2} + \frac{\mathrm{M}}{2\,(d^2+\frac{1}{2}l^2)^{3/2}},$$

where M is the magnetic moment of the specimen, d is the distance between magnetometer needle and centre of specimen (17·93 cm.) and l half the distance between the poles in the specimen (4 cm.).

The length of each of our specimens was 10 cm., and for this the field due to unit magnetic moment was

$$\Gamma = \mathrm{H/M} = 2\cdot663\times10^{-4}.$$

In using the magnetometer the routine has been to set it up on A just before—on B and C halfway through—and on D just after a set of observations of the Richardson effect; and at each position, besides measuring the deflection of the needle due to the specimen, to standardise the instrument by sending a current from a standard cell through 15,000 ohms in series with the coils U U; the specimen being demagnetised.

The coils U U of the magnetometer were small—about 10 cm. diameter and 5 cm. apart—and we therefore determined the strength of the field they produced by setting up the magnetometer between two much larger coils (mean circumference 87·8 cm., distance apart 101·25 cm., number of turns on each 25) and alternately measuring the deflection of its needle due to a reversal of a current of 22·63 m.amps. in these coils and of the current used for standardising in U U.

The means of several such deflections were respectively 6·52 cm. and 3·925 cm. on the scale; from which it follows that the field produced by the magnetometer standardising current was $5\cdot768\times10^{-4}$ and the magnetic moment of the specimen in any measurement of the R effect

$$\mathrm{M} = \frac{5\cdot768\times10^{-4}}{2\cdot663\times10^{-4}}\times\frac{\phi_1}{\phi_2} = 2\cdot166\times\frac{\phi_1}{\phi_2},$$

where ϕ_1 is the steady deflection of the magnetometer needle due to a reversal of the

* For the ordinary end-on and side-on positions of Gauss the errors would have been 2·4 per cent. and 1·7 per cent. respectively, for the same error in l.

magnetism in the specimen, and ϕ_2 that due to a reversal of the standardising current in U U.

It is important that the process of magnetising the specimen should be over in a time which is very short compared with its free period. For this purpose a key was designed which is shown in fig. 1; p and q are pivots about which turn the heavy bars pP and qQ of $\frac{1}{2}$ inch square section brass. These bars, when they come into contact at the platinum pieces P Q close the circuit of the helix.

In setting the key for use qQ is first held up by a small catch W; pP then rises under the pressure of the spring shown below it, but stops short of contact with Q by the spring carried above pP coming against the fixed platinum contact O. On now releasing the catch Q falls against P, thereby closing the helix circuit, and the latter is then immediately opened again by the breaking of the contact at O. To avoid jar Q is mounted on a short stiff spring, not shown.

It will be seen that the time interval between closing and opening the circuit is determined partly by the height from which qQ falls, partly by the position of the contact O, and partly by the position of the stop R carried by pP; as R must come in contact with the upper spring before the break occurs. Adjustment of the time interval may thus be obtained by alteration of the positions of R or O or of the height of the catch which holds up qQ. In part of our work we used a time interval of $6\cdot0\times10^{-2}$ second, and in the rest one of $5\cdot3\times10^{-3}$ second. The change had no obvious effect on the values of the ratio obtained. When the contacts are clean and the movements of the bars quite free, consistent results are obtainable with this key.

The following data may be added in completing the account of the apparatus:—

The magnetising helix contains 1290 turns of No. 17 double cotton covered wire, wound in six layers from the slide rest of a screw cutting lathe to obtain uniformity; a sheet of thin cardboard separating each layer from the next.

Length of helix, 39 cm. Diameter of brass tube on which it was wound, 6·35 cm. Diameter of tube E, 3·8 cm. Length of E, 44 cm. Effective length of beam of light by which scale S_1 is seen, 61·3 cm.

Observations and the Elimination of Errors.

When a uniform cylinder of iron is suspended by a fibre which forms a continuation of its axis in an exactly uniform and vertical magnetic field, the angular momentum imparted to it by a sudden change in the value of this field is due to the Richardson effect alone, and is given by

$$I\omega = \theta_0\sqrt{Ic}, \qquad (1)$$

where ω is its angular velocity directly after the change is completed,

c the torsional rigidity of the fibre,

I the moment of inertia of the suspended system,

and θ_0 the angle through which the latter is turned by the impulse, after correction for damping.

This may be written

$$I\omega = \theta_0 t_0 c/2\pi, \quad (2)$$

where t_0 is the time of swing of the system, also corrected for damping.

If these conditions were exactly obtainable it would thus be an easy matter to measure the Richardson effect; but they are not. And as they imply the balancing of forces which are very large compared with those to be measured (*e.g.* the momentary action of the magnetising field or the sustained action of any unneutralised remainder of the earth's field upon the specimen) it is easy completely to mask the effect sought by errors due to imperfect adjustment, and elimination of these errors thus comes to constitute by far the most important part of the work of measurement.

Error due to earth's field.—The magnetic axis of the specimen is not in general exactly vertical. When, therefore, the magnetic moment of the specimen is altered its small horizontal component, m, is altered too, and any existing horizontal field, f, which is not parallel to m will alter the position of equilibrium of the specimen. The error thus introduced is avoidable by first adjusting m parallel to f; but its magnitude and consequently the perfection required in the adjustment may be much lessened if f and m are reduced.

To reduce f a rectangular coil 4 ft. high and 3 ft. wide was set up with its plane vertical and at right angles to that of the meridian, and its centre coincident with that of the specimen; a certain amount of rotation being possible about its vertical axis. The four corners of the coil could be clamped to rigid supports for any required position of the coil; and a fine adjustment was then obtainable by horizontal screws bearing against its vertical sides at right angles to its plane, the sides being sufficiently slender to bend under pressure of the screws.

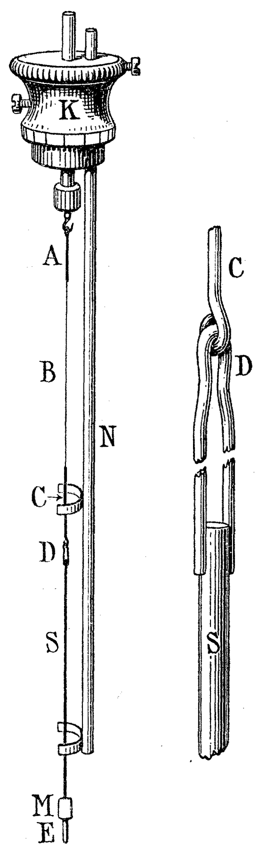
A steady current of about 0·2 amp. through this coil almost neutralised the earth's horizontal field, and what was left could be altered in either direction or magnitude by turning the coil or adjusting the current in it with wire rheostats. The difference between the times of swing of the specimen when magnetised in opposite directions was taken as an indication of the degree to which f was still unneutralised.

To reduce m the magnetic axis of the specimen was made to hang more vertically by causing the latter to tilt slightly. This tilt, which was of known amount and under control, was brought about as follows.

In fig. 2A is given a drawing of the specimen, S, and its suspension. A is a small wire hook to which the quartz fibre B is cemented by two specks of shellac; the second (upper) speck making it possible to melt the lower one with the specimen hanging in position, and so to limit the stress in the fibre to one of pure tension where it joins the hook. C is a second hook cemented similarly to the other end of the fibre, and linked with a small non-magnetic wire loop D soldered to the top of the specimen. It is by a bend at this link (shown about ten times magnified at fig. 2B) that the adjusting tilt referred to above is introduced; the link itself being filled with paraffin wax which can be melted for making the adjustment and which, when cold, locks the joint.

M is the mirror, about 2 to 3 mm. wide and 3 to 4 mm. high and E a second loop, also non-magnetic, soldered to the specimen, to which M is attached by soft wax.

The gallows, used for introducing the bend, is shown diagrammatically in fig. 3. It consists essentially of three metal V's, I., II., III., supported on a rigid frame (not shown) of which I. and III. are fixtures for a given specimen, but capable of both vertical and horizontal adjustment; and the two sides of II. are separately adjustable in two horizontal directions at right angles by means of the micrometer screws H H which work through rigid supports.



Figs. 2A and 2B.

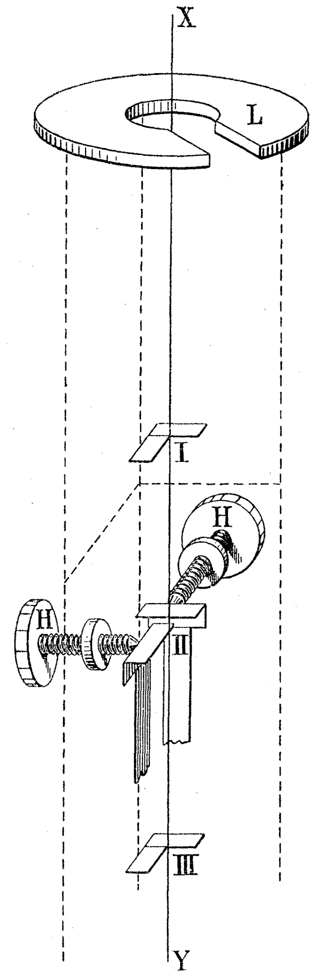


Fig. 3.

I., II., III. are vertically over one another on the line X Y, and centred on this line above them is the platform L into which fits the torsion head that carries the fibre (fig. 2A). The specimen can thus be suspended in front of the V's, and by tilting the whole gallows backwards through about 60 degrees made to lie in the V's with the fibre kept taut by its weight and with the same face uppermost.

It is thus possible by altering H H and melting the wax in D to introduce any desired bend into the joint in either of two vertical planes at right angles, and the effect of

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

this on the tilt of the specimen may be tested by letting the latter hang freely and applying horizontal fields in succession at right angles to these planes.

The pitch of the H screws is 0·8 mm., and we found it possible to adjust to about the hundredth part of this distance with fibres of average strength. The result was that we were able to work throughout with complete reversals of the magnetism instead of being confined to the demagnetising part of the magnetic cycle as STEWART was. It is possible that the consequent gain of symmetry may be connected with the more perfect agreement of our values among themselves.

Having made f and m as small as possible the adjustment of m to parallelism with the horizontal field was taken as complete when the mirror faced in the same direction after the magnetism was reversed as before. It took some time to reach this state, however, and as minute variations in the conditions were constantly upsetting it, we usually found it advisable in practice to take readings of the Richardson effect* for a number of very nearly adjusted cases, of which some were accompanied by a small positive and the rest by a small negative shift of the zero. The readings were then either plotted as ordinates with the corresponding shifts as abscissæ and the ordinate at the point of intersection of the resulting curve with the line of zero shift taken as the value corresponding to adjustment for exact parallelism, or, when the zero shifts were sufficiently small, the simple mean of all readings was used instead ; care being then taken that there was, on the whole, as much zero shift among the readings in the positive direction as in the negative.

Error due to field of helix.—If the magnetising field and the magnetic moment of the specimen possess horizontal components which are not parallel to one another, the specimen will receive an impulse when the helix current passes which may either help or oppose the R effect. There are here two distinct cases to consider. The magnetism acted on may be that which is produced by and reversed with the helix current. The impulse is then in the same direction whichever way the specimen is magnetised, and error due to it may consequently be eliminated by taking the mean of the apparent values of the R effect for equal and opposite magnetic changes.

Or the magnetism acted on may be permanent, in which case the impulse always either helps or opposes the R effect, no matter which way the specimen is magnetised. There are two ways of eliminating this error. One is to turn the specimen through 180 degrees and repeat the measurements ; the other to put on the helix current several times in the same direction, so as to get the magnetism up to the highest value possible with that current, and then to observe the throw of the specimen when the same current is again put on. This throw (referred to in what follows as the helix effect) will be due only to the mechanical action of the current on the specimen as the R effect is now absent ; and the difference between the helix effects for + and − magnetism divided by 2 will therefore be the correction to apply for the effect of permanent magnetism.

* Referred to in what follows as the R effect.

In this connection it may be mentioned that we made no attempt to neutralise the vertical component of the earth's field in the helix. The presence of this field is equivalent to the setting up of a small permanent magnetic moment in the specimen, and as both the above methods of avoiding error from permanent magnetism were applied in our work, we felt that there was little to be gained by going to the elaboration of removing it, especially as its removal would entail the possibility of fresh error from variations of the current used to remove it.

The above may be called the direct helix errors; but there is also an indirect way in which helix error may arise. Besides the simple swing about the fibre as an axis, the helix may cause the specimen to oscillate as a pendulum about the top of the fibre, or more quickly about its own centre of gravity between extreme positions that form an X with one another. Both these types may set up secondary oscillations about the fibre as an axis which will be superposed on the throw set up by the R effect.

These secondary oscillations may vary from the merest quiver which slightly blurs the scale divisions to a long period swing which may be large enough to set the scale dancing so vigorously as to render observations impossible. But we found that by a small tilt of the helix in the right direction we could in general reduce these disturbances to small dimensions compared with the throw being measured; and as their period was also small compared with that of the throw it was often possible to observe them execute a complete oscillation at what was practically the end of the throw.

Under these circumstances the true end of the throw and the middle of the oscillation sensibly coincide, so that by reading to the latter point instead of to the furthest distance reached by the cross wire, the error introduced by the oscillation could be eliminated.

It was in this way that many of our observations were made; but there were cases in which it was not easy to see the inner limit of the oscillation, and where consequently its outer limit was all that could be obtained. The plan was then to adjust the helix effect by tilting the helix until it overpowered the R effect and caused the throws for both + and − magnetisation to be in the same direction in spite of the presence of the R effect. Each throw was then altered by the oscillation by about the same amount, and their difference divided by 2 was consequently the correct value of R so far as error due to the oscillation was concerned.

Of these two methods of eliminating the oscillation error the second is obviously to be preferred, but it is, unfortunately, not always available. For while it is of primary importance that the oscillations be small, this fact limits the range of possible tilting of the helix to an angle so small that it does not always include tilts for which the helix effect is sufficiently large.

In our earlier work the method of reading to the centre of the superposed oscillation was used, our aim being to keep the helix effect low so that the positive and negative R effects were more or less equal on opposite sides of zero; and it was not until we encountered a specimen, which for some reason refused to be adjusted unless the helix effect was large, that we came to realise the value of this condition. Since then, as it has happened, we have been able to use the second method in all our work.

Another method of obtaining a large helix effect, but without tilting the helix, is to employ a magnetising current which is greater than that required for magnetic saturation.

On setting up the specimen.—The ends of the quartz fibre having been respectively attached to the specimen and the wire hook A (fig. 2A) the specimen was inverted and hung by the loop at its lower end. To the wire hook, now at the bottom of the fibre, was next attached the brass cylinder of known moment of inertia, in terms of which the torsional rigidity, c, of the fibre was determined by oscillation.

In the case of strong fibres a brass cylinder of diameter 0·836 cm., and mass 1·240 grm. was employed, its axis being vertical; but with weak fibres it was necessary to use a lighter cylinder, and particularly one of smaller diameter, to avoid excessive damping. The diameter of this was 0·26 cm. and its length 1·2 cm. And as in the case of such a long slender cylinder it was not safe to depend on its geometrical axis coinciding with the axis of turning, its moment of inertia was obtained in terms of that of the heavier cylinder by measuring their periods when hung in succession upon the same fibre. The agreement with the value obtained from the dimensions of the small cylinder was, however, very close.

As a check on the value of I for the large cylinder the period of a third cylinder (mass=1·250 grm., diameter = 2·023 cm.) was taken upon the same fibre and used to determine I for the first. The two values agreed to within 0·1 per cent.

c being known, the specimen was next suspended right way up from the torsion head, and after being "gallowsed" was transferred to the magnetising helix, where its own moment of inertia could be obtained in terms of c and its period of oscillation, t.

In all cases the true period, t_0, corrected for damping was calculated from the observed period, t, by the usual formula

$$t_0 = \frac{t}{\left(1 + \frac{\delta^2}{4\pi^2}\right)^{\frac{1}{2}}},$$

where δ is the Napierian logarithm of the ratio of successive swings on the same side of zero.

Routine of the observations.—The magnetic disturbances due to the starting and stopping of the electric trams were found to be sufficient to keep the magnetometer needle moving over several millimetres in the day time, and we were therefore obliged to work at night; but it was fortunately just possible to obtain the data for one complete determination of the Richardson ratio between 11.45 p.m. and 5.45 a.m. when the trams were not running, and all our measurements have therefore been made in that interval.

In Table I. are given the figures and calculations for one complete determination of R (that for specimen No. 14) each value of θ being the mean of more than twenty separate observations.

TABLE I.

	Mean zero shift.	θ_1.	Helix effect.	θ_1, corrected for Helix.	k_1/k_2.	t.	Magneto-meter.
	cm.	cm.	cm.	cm.		sec.	
0° $m+$	$-0\cdot002$	$+0\cdot728$	$+1\cdot206$	$-0\cdot478$	$1\cdot495$	$5\cdot22$	A. $1\cdot81$
$m-$	$-0\cdot001$	$+1\cdot301$	$+1\cdot088$	$+0\cdot213$	$1\cdot48$	$5\cdot19$	B. $1\cdot83$
180° $m+$	$+0\cdot001$	$-1\cdot228$	$-1\cdot037$	$-0\cdot191$	$1\cdot45$	$5\cdot15$	C. $1\cdot84$
$m-$	$0\cdot000$	$-0\cdot588$	$-1\cdot004$	$+0\cdot416$	$1\cdot48$	$5\cdot25$	D. $1\cdot71$
Means	—	—	—	$0\cdot324$	$1\cdot476$	$5\cdot20$	$1\cdot798$

Scale distance from mirror for θ_1 . . . . $61\cdot3$ cm.
$1+\lambda$ corresponding to k_1/k_2 . . . . $1\cdot197$
Damping divisor for t . . . . $1\cdot008$
c . . . . $1\cdot743\times10^{-4}$

$$I\omega = \frac{1\cdot743\times10^{-4}\times0\cdot324\times1\cdot197\times5\cdot20}{8\times61\cdot3\times1\cdot008\times\pi} = 2\cdot262\times10^{-7}$$

$$M = 1\cdot798\times2\cdot166 = 3\cdot895.$$

$$\text{Ratio} = \frac{2\cdot262\times10^{-7}}{3\cdot895\times1\cdot13\times10^{-7}} = 0\cdot514.$$

The procedure was as follows: the specimen being suspended in its initial position (0 degrees in the Table) and the earth's horizontal component so nearly neutralised that the permanent change of position of the cross wire of T_1 on S_1 (fig. 1) was not more than two- or three-tenths of a millimetre when the magnetism was reversed, a series of consecutive magnetic reversals was made in alternately opposite directions and for each of these θ was observed. These reversals are marked respectively $m+$ and $m-$.

The specimen was next rotated through 180 degrees by turning the torsion head and a second set of magnetic reversals made (180°, $m+$ and $m-$ in the Table).

θ_1 was thus observed under four separate sets of conditions, corresponding to the four lines in the Table. The values for each of these were divided into three groups with which were sandwiched four groups of observations of the "helix effect" (see above). The means of the latter are given in Table I., under "helix."

Their values are large—over a centimetre on the scale—the case chosen for illustration being one of those in which the R effect was overpowered by that of the helix for the purpose of causing both θ_1 throws to be in the same direction (see above). The true sign of the R effect is that of the observed θ_1 after subtracting from this the corresponding helix throw; this difference being given under "θ_1 corrected for helix."

It might have been expected that correcting for the helix effect in this way would have brought the values of θ_1 for $m+$ and $m-$ to equality. That it has not done so is due to the fact that the helix effect as measured is greater than that which accompanies the R effect, because in the case of an R effect the helix acts on magnetism which is reversing, and its effect is due to the difference between two opposing forces, whereas, when measured directly it is due to a force in one direction only. It will be seen from the figures that a smaller value of the helix effect is in fact required to bring the corrected values of θ_1 to equality. There is no error arising from this cause, however, when all four values of θ_1 are taken into account.

The signs of m are connected with those of θ_1 (corrected for helix) by the rule that a $+$ sign before θ_1 means a rotation of the specimen in an anti-clockwise direction when seen from above.

Thus the first line in Table I. corresponds with $m+$, a change of upper pole from N to S, which caused the specimen to rotate in a clockwise direction seen from above $(-\theta_1)$, that is to say, in a direction opposite to that of the electrons, as it should do.

The column headed k_1/k_2 contains the ratios of pairs of successive deflections in opposite directions, from which the damping factor is determined. The directions of these deflections were such that the cross wire moved in the same direction in going from k_1 to k_2 as it did in the initial throw due to R. In the bulk of the measurements the deflection to k_1 was obtained from a small coil near the specimen in which a current could be started and stopped by a key.

If θ_0 denotes any throw corrected for damping and θ_1 the observed (uncorrected) throw corresponding, we have used as damping factor the quantity

$$1+\lambda = (k_1/k_2)^{\frac{1}{\pi}\arctan\frac{\pi}{\log_e(k_1/k_2)}}$$

such that

$$\theta_0 = \theta_1(1+\lambda).$$

The column headed t gives the period of the specimen obtained by observing the passages of the cross wire past its zero position on the scale.

The difference between the values of t for $m+$ and $m-$ is due to the unneutralised portion of the earth's horizontal field; and as it was not as a rule possible to reduce this exactly to zero we examined its influence on the measurements by allowing the difference between the two values of t to become abnormally high.

In the three cases tested they differed by 30, 23 and 64 per cent. respectively. The corresponding points, which are to be found on Curve 1A, plotted at the values of $M = 3\cdot63$, $3\cdot86$ and $7\cdot26$, will be seen to fall on that curve quite as closely as on any of the others, and as in the bulk of our work the two values of t never differed by more than a few units per cent., it is safe to conclude that its accuracy cannot have been appreciably influenced by imperfect neutralisation of the earth's horizontal field.

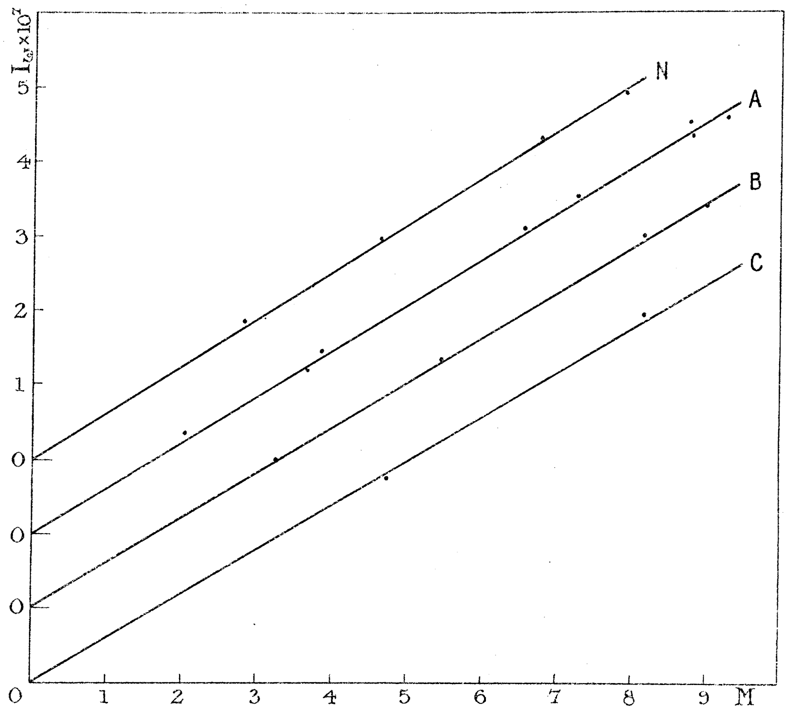
Lastly, in the column headed magnetometer, the values of the ratio ϕ_1/ϕ_2 (see above) are given.

To show the kind of agreement existing between the numbers of which the means are given in Table I. we add the record of the actual observations from which the third line of that table is calculated (Table II.).

TABLE II.—One quarter of the record of a complete determination of R.

Magnetometer C.	Helix.	Zero shift.	θ_1.	k_1.	k_2.	$4t$.	
						+	−
	cm.	cm.	cm.	cm.	cm.	sec.	sec.
	−1·06			3·02	2·07	20·2	20·9
Standardising	−1·055			3·57	2·47	20·7	20·2
		+0·015	−1·165				
Current.	−1·045			4·12	2·86	21·0	20·8
		+0·01	−1·25				
	−0·94			2·93	2·00	20·7	20·3
		+0·02	−1·185				
+ −	−1·13			3·21	2·20	20·6	20·7
		+0·005	−1·275				
15·55							
11·00		(−0·03	−1·29)				
15·54							
10·98		(−0·03	−1·365)				
15·54							
10·95		−0·015	−1·205				
15·53	−1·08						
10·95		−0·01	−1·24				
15·51	−1·045						
		−0·015	−1·31				
	−1·125						
15·53 10·97	−1·075						
		−0·03	−1·25				
4·56	−1·055						
		0·00	−1·35				
		+0·005	−1·35				
		−0·005	−1·115				
$m+$ $m-$							
		0·00	−1·285				
12·75							
21·09		+0·005	−1·11				
12·72							
21·11		−0·005	−1·38				
12·71	−0·94						
21·12		−0·01	−1·16				
12·71	−0·98						
21·11		−0·005	−1·20				
12·69	−1·075						
	−1·005						
		+0·005	−1·255				
12·72 21·11	−1·01						
		(+0·025	−1·315)				
8·39							
		0·00	−1·17				
	−0·965						
		+0·02	−1·26				
	−1·03						
		0·00	−1·055				
$\frac{8\cdot39}{4\cdot56} = 1\cdot84$	−1·025						
	−1·04						
	−1·055						
	−1·037	−0·001	−1·228	3·37	2·32	20·61	

The bracketed readings are ruled out of the means, our plan being to discard those for which the zero shift was 0·3 mm. and over, and also the largest of that sign of shift which preponderated, if necessary for bringing the average shift sufficiently near to vanishing. For the method (already explained) of obtaining θ_1 for no shift by plotting θ_1 with the shift is not accurate when the shifts are very small, as the variations in θ_1 make it difficult to trace the line of the resulting curve; and it is better then to take the arithmetical mean of θ_1 for shifts that vanish on the average as has been done in the present instance. It is true that the line when traced shows curvature which is not allowed for by taking the mean in this way, but the curvature is the same for the curves of both $m+$ and $m-$, and its effect on θ_1 consequently vanishes when the values of θ_1 for both signs of magnetism are combined.



Curve I.

Discussion of Results.

The theoretical value of the Richardson ratio for gyrating electrons is $1 \cdot 13 \times 10^{-7}$. In what follows the term Ratio with a capital R is used to denote the proportion between the value of the Richardson ratio experimentally obtained and $1 \cdot 13 \times 10^{-7}$.

After setting up the apparatus we started by making a few rough determinations of this Ratio with different iron wires, the results from which lay between 0·5 and 0·6. They were found to vary among themselves, however, and the first thing to be done was, therefore, if possible to trace their variations to one or more of the physical conditions under which they occurred.

For this purpose a specimen (No. 10) that had behaved well in the helix was chosen, and its detailed examination undertaken, beginning with its degree of magnetisation (length of No. 10, 10 cm.; diameter, 0·036 cm.).

It will have been understood that the magnetism we worked with was that left in the specimen after the passage of a momentary current through the helix; the R effect being produced by the reversal of this residual magnetism following a second passage of the same current in the reverse direction.

In Curves IA are plotted the corresponding values of angular momentum and change of magnetic moment for a series of residual magnetisations up to the highest the specimen would hold. The straight line passing through the origin and ruled evenly among the individual points shows that the two quantities concerned are closely proportional to one another as they should be; and a second set, B, taken with a higher torsion constant shows the proportionality more strikingly. Set C is for a still higher torsion. Proportionality was also found for the nickel wire, N; the co-ordinates in this case having been magnified five times as the quantities involved were so much smaller than for iron.

Considering that the actual scale deflections obtained for the lowest point on the nickel curve were not much more than a millimetre, the resulting point falls surprisingly near the straight line; but we took about 100 readings and spent two nights in getting it.

The law of proportionality is thus closely established by these measurements, and it implies, of course, that while for all points on one of these straight lines the Ratio is constant, its value changes in passing to one of different slope.

We sought next a possible connection between the Ratio and the torsional rigidity, c, of the suspension. By shortening the fibre at its upper end it was possible to increase c without otherwise altering the nature of the suspension, so that the hang of the specimen remained the same and fresh "gallowsing" was avoided. In this way B was derived from A as well as the still shorter suspension C. The fibre then unfortunately broke and we had to put on a new one, E, chosen to give a higher torsion than C, though its length was about the same as that of A.*

The data for these suspensions are given in Table III. from which it will be seen that the four corresponding values of the Ratio go steadily down in passing from A to E; and, as the last two columns show, tend to a limit very near 0·5 when either λ or the reciprocal of c vanishes.

TABLE III.

	Ratio.	c.	$10^{-4}/c$.	λ.
A.	0·558	$1{\cdot}86\times10^{-5}$	5·4	0·64
B.	0·551	$3{\cdot}92\times10^{-5}$	2·6	0·43
C.	0·535	$6{\cdot}44\times10^{-5}$	1·6	0·33
E.	0·504	133×10^{-5}	0·07	0·07

The possibility of a direct connection between the Ratio and the torsion constant raised an interesting point.

When an impulse is imparted to the suspended system at rest at its centre of equilibrium it produces a throw which is inversely proportional to $\sqrt{c}$. If, on the other hand, a couple is applied continuously the first throw is inversely proportional to c; and even if the couple is not constant its effect will be proportional to a higher power

* This of course meant fresh "gallowsing"; but the fact that this only altered the moment of inertia of the specimen from $1{\cdot}44\times10^{-4}$ to $1{\cdot}55\times10^{-4}$ indicates that the alteration it produced in the hang was small.

of $1/c$ than the square root provided the time its application lasts is not small compared with the quarter period of the system.

From this it follows that if an instantaneously applied impulse is succeeded by the prolonged action of such a couple the effect of the latter on the throw will vanish before that of the impulse when c is increased without limit.

Now the throw of the Richardson effect, which is in the opposite direction to the spin of the electrons after magnetisation, represents a portion of the reaction to this spin; and there was nothing inherently impossible in the idea that another portion might pass through a stage of spinning motion in some part or parts of the magnetic molecules before being transmitted to the rigid structure of the iron in the form of the couple which produces throw.

Thus, if for some reason half the reaction appeared at once as such a couple on reversing the magnetism, and the remainder only after the dying down of an intermediate spin stage, we should expect to find the Ratio tending towards 0·5 when c was increased without limit, much as it does in Table III.

Further, the fact that Curves 1 are straight lines passing through the origin implies that the difference between the momentum ordinates for any two of these lines, measured at the same value of the magnetic moment M, is proportional to M; and this proportion might also be expected on the above hypothesis.

It looked, in fact, as though we might have found in this way the explanation of the low value of the Ratio, until the application of the two following tests. The first depends on the fact that an increase of the moment of inertia of the suspended system decreases the relative importance of the initial impulse in producing throw, and should therefore increase the Ratio on the above view. After C in Table III., and before the fibre broke, we attached a small wire loop to the bottom of the specimen (D, Table IV.). This increased its moment of inertia from $1 \cdot 4 \times 10^{-4}$ to $7 \cdot 9 \times 10^{-4}$, but instead of increasing the Ratio it reduced it from 0·535 to 0·515.

The second test depends upon the behaviour of the specimen after the completion of its first throw. The values of the Ratio actually obtained are never much above 0·5, which shows that if the missing momentum is to be accounted for in this way only a small fraction of it makes its appearance during the first throw, and the remainder is therefore to be expected later. In other words, if θ_1 and θ_2 are the limits of the first and second throws after the magnetic reversal, the effect of this remaining momentum will be to decrease θ_2 (numerically) no matter what the direction of θ_1; and a damping constant calculated from θ_1 and θ_2 will consequently always be too large.

We give below, under "Damping Anomalies" a detailed examination of the data available for this calculation. From this it appears that the resulting damping constant actually is greater than that obtained from k_1 and k_2, but the difference is very small—not more than about $\frac{1}{2}$ per cent. Also that there is good reason to suppose it unconnected with any gradual appearance of momentum in the specimen, though the evidence for this is not quite conclusive.

TABLE IV.

Specimen.	Ratio.	θ_1 mm.	$1+\lambda$.	t_0.	$c\times10^5$.	$I.\times10^4$.	n.
10 A.	0·558	5·83	1·640	17·51	1·86	1·43	320
10 B.	0·551	4·51	1·435	11·96	3·92	1·42	160
10 C.	0·535	3·75	1·330	9·24	6·44	1·40	80
10 D.	0·515	2·13	1·300	22·10	6·44	7·90	50
10 E.	0·504	1·25	1·071	2·11	133·0	1·50	80
10	0·523	1·00	1·077	5·47	133·0	10·1	150
10	0·566	12·30	1·647	18·60	1·77	1·55	30
10	0·563	8·45	1·752	24·20	1·77	2·63	30
10	0·541	8·31	1·707	24·55	1·77	2·69	40
10	0·524	4·02	1·714	48·60	1·77	10·60	40
10	0·514	4·04	1·125	3·96	40·50	1·60	50
10	0·530	0·99	1·772	11·03	40·50	12·60	120
10	0·580	1·21	2·153	8·12	40·50	6·73	50
10	0·589	1·20	2·103	8·30	40·50	7·04	50
10	0·532	1·32	1·872	16·08	22·3	14·6	80
10	0·520	1·31	1·874	16·07	22·3	14·6	150
10	0·504	1·19	1·827	17·63	22·3	17·6	80
10	0·512	6·42	1·168	4·94	22·3	1·46	60
10	0·497	6·42	1·17	4·97	22·3	1·40	70
10	0·558	3·26	1·857	6·87	22·3	3·63	150
10	0·528	1·90	1·29	16·21	22·3	14·8	80
10	0·521	3·84	1·29	7·90	22·3	3·53	90
10	0·486	1·03	1·16	8·66	78·4	14·9	110
S	0·494	2·30	1·11	4·83	47·9	2·84	220
S	0·530	0·91	1·904	7·97	47·9	7·69	140
11	0·519	2·31	1·125	3·56	33·6	1·07	110
11	0·532	2·09	1·524	4·18	33·6	1·50	50
14	0·514	3·24	1·197	5·15	17·4	1·17	90
N	0·570	2·60	1·956	29·4	0·72	1·57	280
N	0·525	1·00	1·151	4·67	26·6	1·47	130

Of the two tests the first thus shows that we cannot explain the fall of the Ratio in Table III. by the gradual appearance of momentum in the iron during the first quarter period after magnetic reversal; and the second makes it almost, though not quite, certain that neither does any momentum appear during the remainder of the period.

This result does not, however, negative the possibility that some of the reaction may take the form of spin. If part of the reaction fails to reach the rigid structure of the iron in time to be measured in θ_1 this may be due to a break, more or less complete, in some of the paths along which momentum might otherwise pass immediately to the iron from the rotating electrons when their orbits are turned. Such a break may be pictured as a place where slipping occurs in the mechanism which constitutes the momentum path, and slipping has in fact been suggested* (though without much conviction) as one among other possible causes of the low value of the Ratio.

* O. W. RICHARDSON. Solvay Congress, 1921.

Now, if in its passage the momentum reaches matter which is free to spin, it is likely to pass into the spinning form; with the result that it ceases to travel, and we have an illustration of the sort of way in which slip in the momentum paths might occur.

All that our experiments show is that, if such slip is really present, the corresponding spin is either of indefinite duration or at least lasts for many minutes before it dies down.

The nucleus, or some part of it, might in that case play the part of the body free to spin; but there would, of course, be no question of any magnetic effect due to its motion. Its mass is so large compared with that of the electrons that its velocity, and therefore its magnetic moment, would be negligible compared with theirs. The magnetism which enters the Ratio would thus, for practical purposes, be that of all the electrons responsible for the process of magnetisation.

On the other hand, these electrons might be expected to behave in two distinct ways as regards the reactions to their own momentum; those concerned with the rigidity of the material passing their reaction directly into the iron as a whole, and those more closely associated with the nucleus causing the latter to spin; with the result that only the first of those reactions would be measured and the Ratio would consequently be too low. We know so little of the internal arrangements of atoms that more detailed speculation on this matter is not likely to be profitable at present.

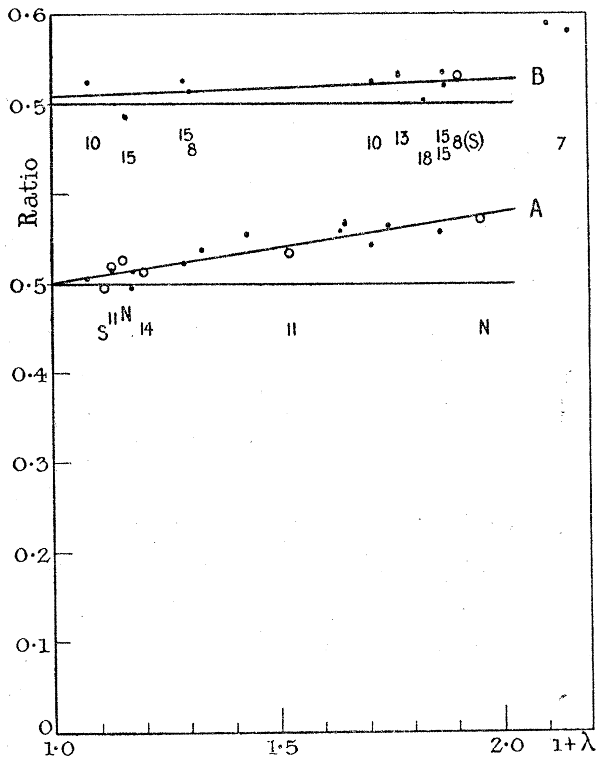
For an explanation of the fall of the Ratio in Table III., we were thus thrown back on the alternative of a possible connection between the Ratio and the damping coefficient. We therefore started a long series of determinations with the same iron specimen (No. 10), in which the damping was varied between wide limits by alteration of the values of c, I, and λ, by means of paper or mica vanes, or weights attached to the lower end of the specimen, or by alterations in the suspending fibre. The results plotted in Curves II, are given by the black dots.

Along with these are plotted the Ratios for three other iron wires, 11, 14 and S, as well as for a specimen of Nickel, N. These are shown by circles, and may be identified in Curve A by the numbers and letters below them. In Curve B the only circle that appears stands for another determination of the Ratio for S.

The data from which these curves have been plotted include the whole of our work, and will be found in columns 2 and 4 of Table IV., where A - - - E have the same meaning as in Table III. There has been a certain amount of grouping of the data for this plotting. Each line in Curves I. is represented by a single point in Curves II., and where the initial throw was less than a millimetre we have plotted the mean values of the Ratio for two or three separate determinations, except in cases where the observations have been more numerous than usual. The approximate number of separate readings of θ_1 used in each Ratio determination is given in column n, Table IV.

The multiplication of observations will be seen to bear out the connection between λ and the Ratio suggested by Table III.; but in addition to this it appears that the results are to some extent dependent on the moment of inertia of the suspended system; a larger inertia tending to bring the Ratio nearer to $0\cdot5$ for a given value of damping.

This is seen by a comparison of the two Curves A and B. I for an unloaded specimen always lay between $1 \cdot 0 \times 10^{-4}$ and $1 \cdot 6 \times 10^{-4}$; and the Ratios for the unloaded condition are all plotted in A, as well as those for five rather higher values of I lying between $2 \cdot 6 \times 10^{-4}$ and $3 \cdot 6 \times 10^{-4}$.



Curve II.

Curve B contains all the higher inertias with which we worked (7×10^{-4} to 18×10^{-4}) the values of I for each point multiplied by 10^4 being given below it. The rise of this curve with increase of damping is much less rapid than that of A until $1 + \lambda$ is above $2 \cdot 1$* when the Ratio jumps up to a value comparable with the higher Ratios of A.

Neglecting this point as being outside the range of our experiments, it may be said that, roughly speaking, the Ratio is a linear function of the damping, but one that alters when I is altered. So far as damping goes, its effect on the Ratio should vanish for $1 + \lambda = 1$. We therefore ruled the lines which lay most evenly among the experimental points for each curve, giving weight in the case of A to both dots and circles† since they have presumably all been influenced by the damping effect, and leaving out of account both S and the two high points in the case of B. These lines cut the Ratio axis at $0 \cdot 502$ for A and $0 \cdot 510$ for B.

The data for Curve A are much more dependable than those for Curve B, as the throws

* For this value of $1 + \lambda$ the ratio θ_1/θ_2 is about 19, and the motion consequently almost dead beat.

† Except S, as there may have been another disturbing influence at work there besides the damping. See *Eddy Currents* below.

obtained in the case of B had been rendered very small by the increase in the inertia of the suspended system (see Table IV.). The difference of $1\frac{1}{2}$ per cent. between the two numbers is, however, well within the limits of experimental error, and they may therefore be regarded as coinciding for practical purposes, their weighted mean being nearer 0·502 than 0·510—say, 0·505.

The fact of their coinciding is important because it shows that at this point the value of the Ratio has ceased to be influenced by the inertia as well as by the damping; so that we may take it as affording a value of the Ratio which is independent of both these sources of error.

0·505+0·002,* a number which is indistinguishable from one-half, is thus the value obtained for the Ratio in the case of Specimen No. 10, when all errors known to us have been eliminated.

The way in which the circles fall among the dots in A strongly suggests that the deviations of all these points from the one straight line are no more than accidental. It was therefore interesting to calculate the mean deviations, from the line, of the points belonging to each specimen separately. By adding or subtracting the deviation of each point to or from 0·502, according as it falls above or below the line, taking the mean of the results and adding 0·002,* we obtain values of the Ratio for each specimen which take account of all the observations made upon that specimen.

The results are given in Table V., under "Ratio," and show an agreement to about $1\frac{1}{2}$ per cent. between the highest and lowest Ratios if for No. 10 we take the mean value given above.

TABLE V.

Curve.	Specimen.	Diameter.	Ratio.	Number of points.
		mm.		
A.	Iron No. 10	0·36	0·504 } 0·507	12
B.	,, ,, 10	0·36	0·512 }	9
A.	,, ,, 11	0·28	0·501	2
A.	,, ,, 14	0·29	0·500	1
A.	Nickel	0·32	0·505	2

Nos. 10 and 14 were the soft wire used by florists.
No. 11 much harder steel sonometer wire.

In the light of these figures it seems almost impossible to doubt that the Ratio is 0·5 for ferro-magnetic substances. The experiments on the Heusler alloys, if successful, will be particularly interesting from this point of view.

* The Ratios given in Table V. are greater than those obtained from Curve II. by 0·4 per cent., the difference representing a correction explained below at the end of the section on *Damping Anomalies*.

Origin of the Slope of Curves II.

So far as the determination of the Richardson Ratio was concerned the results of Table IV. completed the work and appeared to justify our use of this modified form of the ballistic method ; but we felt that the effect of inertia on the slope of the curve needed further examination to determine, if possible, whether it had its origin in the particular physical conditions of our experiments, or was simply due to some imperfection in the mathematical correction for damping.

A copper wire of dimensions approximately the same as one of the iron specimens was, therefore, fitted up with a minute magnet, consisting of about 1·5 mm. of fine sewing needle fixed horizontally to it near its lower end, and a paper vane of height 2·6 cm. and width 1·1 cm. attached below it.

Of this vane a strip at the bottom about 2 mm. wide was bent up to form a small trough in which a short non-magnetic wire could be laid horizontally in order to change the moment of inertia of the suspended system without altering the shape of its outer surface. The whole system was then placed inside the helix as in an ordinary determination of the R effect, but instead of sending the momentary current through the helix, it was sent through a vertical coil fixed near the helix and arranged to deflect the magnetic needle.

In this way the ballistic effects for various dampings could be obtained without the complication of magnetic changes ; the angular momentum being equal to $m\mathrm{Q}\Gamma$, where m is the magnetic moment of the small needle, Q is the quantity of electricity that passes through the deflecting coil, and Γ the field at m due to unit current in the coil.

Without going to the elaboration of determining these quantities absolutely we were able to keep the product $m\mathrm{Q}\Gamma$ constant. Hence, since c remained the same, $\theta_1 (1+\lambda) t_0$ should also be constant ; and by calculating λ from θ_1 and θ_2 we could determine how nearly this was the case.

Two sets of observations were made for values of I corresponding to the inertia conditions of Curves A and B respectively. In each, observations with the non-magnetic wire load present were sandwiched with others in which it was absent from the paper trough ; the damping in the loaded state being, of course, less than that in the unloaded.

In this way values of $\theta_1 (1+\lambda) t_0$ were obtained for two different dampings in each set of observations, and from these pairs of dampings corresponding values of the Ratio could be found from Curves II. The results are given in the first two columns of Table VI., and in the two last their variations are compared. Thus, in column 3 the number 1·11 is the Ratio 0·569/0·513, in column 4 the number 1·14 is the Ratio 65·4/57·2, and so on.

TABLE VI.

	R. from Curves II. 1	$\theta_1 (1+\lambda) t_0$. 2	3	4
Small inertia—A High damping	0·569	65·4	1·11	1·14
Small inertia—A Low damping	0·513	57·2		
Large inertia—B High damping	0·537	52·5	1·03	1·01
Large inertia—B Low damping	0·521	51·9		

It will be seen that there is fair agreement between columns 3 and 4, better indeed than the data warranted; for though we took a large number of readings of θ_1 in each of the sets, there was much variation among what should have been constant results, attributable partly to imperfect working of the contact key in the deflecting circuit and partly to thermal convection currents in the air, the fibre being necessarily very slender.

We do not, therefore, press these numbers as doing more than show that inertia has much the same effect on $\theta_1 (1+\lambda) t_0$ as it has on R; in other words, that the explanation of the slopes of the A and B Curves is a problem in pure damping unconnected with the special conditions obtaining in measurements of the R effect.

And this is supported by a fact to which Prof. HASSÉ has kindly drawn our attention, that a body oscillating in a viscous fluid drags some of the fluid with it, and that the expression for the effect of this on the motion is of the right type to reduce and, perhaps, to correct the inertia effect we have observed.

Damping Anomalies.

When the damping was not too heavy the second throw, θ_2, and in some cases the third, θ_3, were recorded in addition to θ_1. From these quantities it was, of course, possible to calculate a damping coefficient as well as from k_1 k_2 and the results thus obtained proved interesting.

As already explained, in making a complete determination of the R effect the mean is taken of four separate values of θ_1, corresponding to four separate sets of conditions. Two of these correspond with $m+$ (the production of a N pole at the upper end of the specimen) and $m-$ (N pole below) with the specimen hanging in its initial position; and the other two to $m+$ and $m-$ after the specimen has been turned through 180 degrees about its axis.

If a, b, c, d represent the damping coefficients calculated from the two values of θ (θ_1 and θ_2) for each of these four sets of conditions, the attached scheme gives the connection between the conditions and the coefficients in each case.

	0°	180°
$m+$	a	c
$m-$	b	d

The four coefficients were found to differ considerably among themselves, but by amounts which varied much from one determination of the R effect to another; and as the damping coefficient depends on the controlling force to which the suspended system is subjected, and part of this force is due to the very variable amount of the earth's horizontal field which remains unneutralised, we first of all eliminated the effect of this particular cause of variation from a, b, c, d.

Owing to the fact that the damping coefficients calculated from k_1 k_2 were equally affected by this cause the elimination was easily made by reducing the values of a, b, c, d, to what they would have been if the k_1 k_2 coefficients had been constant and equal to their mean value, and it is to these reduced values that reference is made in what follows.

The differences between a, b, c, d (which were found to persist after this reduction had been made) were sometimes positive and sometimes negative; and the first question to be settled was whether they were the result of pure accident or of something more. This was effected by comparing the values of a, b, c, d, calculated from θ_1 and θ_2 with those calculated from θ_2 and θ_3 in ten cases in which both θ_2 and θ_3 happened to have been recorded.

Three types of differences were observed, accompanying :—

(e) the turning of the specimen through 180 degrees alone ;
(f) the turning of the specimen and change of sign of m ;
(g) the change of sign of m alone ;

and of these the difference between $\frac{a+b}{c+d}$ and unity is obviously a measure of e, between $\frac{a+d}{b+c}$ and unity that of f, and between $\frac{a+c}{b+d}$ and unity that of g.

In Table VII. are given the values of these three ratios for the ten cases referred to ; those obtained from θ_1 θ_2, which were greater than unity, having been separated from those which were less for clearness.

TABLE VII.

$\frac{a+b}{c+d}=e.$		$\frac{a+d}{b+c}=f.$		$\frac{a+c}{b+d}=g.$	
$\theta_1\theta_2$	$\theta_2\theta_3$	$\theta_1\theta_2$	$\theta_2\theta_3$	$\theta_1\theta_2$	$\theta_2\theta_3$
1·01	0·94	1·01	1·01	1·06	0·87
1·09	0·85	1·05	0·95	1·02	0·95
1·01	0·93	1·00	0·99	1·04	0·97
1·00	0·98	—	—	1·00	1·00
—	—	1·00	1·05	1·01	1·00
0·99	1·02	0·98	1·00	1·02	0·98
0·89	1·26	0·97	1·00	—	—
0·96	1·10	0·99	1·01	0·99	1·05
0·99	1·01	0·95	1·03	0·94	1·13
0·99	1·02	0·98	1·01	0·99	1·01
0·97	1·00	0·98	1·10	0·98	0·99

It will be seen that in the great majority of cases the variations in each pair of columns are complementary, in the sense that when the $\theta_1\ \theta_2$ value is above 1 the $\theta_2\ \theta_3$ value is below, and *vice versa*. This regularity proves, of course, that the variations are not mainly accidental.

There is, in fact, little doubt that we have in these numbers evidence of the continued play of the disturbing forces met with during the first quarter period of the specimen's motion. These, as we have already explained, are probably mainly due to varieties of pendulum swing, set up by the action of the magnetising current, which, owing to imperfect symmetry of the specimen, set up in turn secondary oscillations about the suspension as axis. The effect of such oscillations upon the main oscillation of the R effect will depend on the relative values of their respective periods and amplitudes.

Suppose the pendulum error to be one of excess (positive) in θ_2 and zero in θ_1 and θ_3. The values of λ calculated from $\theta_1\ \theta_2$ and $\theta_2\ \theta_3$ will be complementary—that from $\theta_1\ \theta_2$ being too small and that from $\theta_2\ \theta_3$ too large.

Next suppose that θ_1 and θ_3 are also subject to pendulum errors, as of course they will usually be. The actual values of λ will now be unpredictable; but since the chances of the error in either θ_1 or θ_3 being positive are equal to those for its being negative there is still the *probability* that they will be complementary; a conclusion which also holds if the error in θ_2 happens to be negative.

It is, of course, necessary for this that these secondary oscillations should persist beyond θ_3, and we found in fact that they did so; their damping coefficients being in general far smaller than that for the oscillations of the R effect.

If, therefore, the numbers in Table VII. are to be taken as indicating the effects of pendulum disturbances, we should expect these numbers also to exhibit the complementary character; and this in fact they do.

We found that the departure of these numbers from unity increased rapidly when the gallows adjustment was imperfectly made; but though we looked carefully for any indication of a connection between the direction of the residual earth's field and the sign of the differences between θ_1 θ_2 and θ_2 θ_3, we failed to find one. This is, of course, what was to be expected if pendulum swing, started by helix current, was their cause—the changes in the sign of the variations actually met with being easily accounted for by changes in the tilt of the helix during adjustment or by alteration in the ratio of the period of the specimen when oscillating about the suspension as axis, to its period when swinging as a pendulum.

There is no difficulty either in finding a satisfactory explanation for the existence of the three types of variations, e, f, g, on the theory of pendulum swings. They follow at once if we suppose that the g swings are started by the vertical component of the helix field, and the e and f swings by its horizontal component, acting in the case of e upon the permanent part of the magnetism in the iron and in the case of f upon the part that reverses.

For consider variation g, and suppose that, as was the case in the early experiments from which these data are taken, θ_1 for $m+$ was in the opposite direction to θ_1 for $m-$. If a vertical helix field produces any horizontal pendulum motion at all it will be in the same direction after a given interval from the start, no matter which way the specimen faces. If it happens to increase a it will therefore increase c, while both b and d will be reduced as the throws for them are in the opposite direction to those for a and c.

The difference between the fraction $\frac{a+c}{b+d}$ and unity (without regard to sign) will thus give a measure of the disturbance due to the vertical field.

In the case of a horizontal helix field the couple producing pendulum oscillations in the specimen is the product of the latter's magnetic moment with the field, but the direction of the secondary oscillations set up will depend also on the direction of the tilt of the specimen from the vertical, an exactly vertical and symmetrical specimen receiving no secondary oscillations at all. Turning through 180 degrees will therefore reverse their direction, but changing the sign of the magnetism will not. Hence, since the throw for $m+$ is opposite to that for $m-$, if a happens to be increased by these oscillations d will also be increased, and b and c will be diminished; $\frac{a+d}{b+c}$ being thus the appropriate fraction in the case of f.

In the same way, it is easy to see that, so far as permanent magnetism is concerned, if the effect of a horizontal helix field is to increase a it will now increase b, while c and d will be diminished. The corresponding fraction for e will therefore be $\frac{a+b}{c+d}$.

To this it may be objected that the permanent magnetism is due mainly to the vertical component of the earth's field, and is therefore small in amount compared with the part that reverses, whereas the variations of e and f are of the same order of magnitude; but it seems possible that this difficulty may be met by the fact that the action of the helix current on the magnetism which reverses is the resultant of two opposing tendencies

—its action on the magnetism initially present in the specimen and its action on the magnetism that succeeds it; while in the case of the permanent magnetism the action is all in the same direction.

As already explained, the process of taking the means of the four values of θ_1 corresponding to a, b, c, d, should eliminate the disturbing effects on θ_1 of the pendulum swings if these are not too large; and it now appears that in the relation of θ_2 to θ_1 we have a means of estimating experimentally the effects of these swings before their dying down has proceeded far. If they cancel one another during the motion of the specimen through θ_1 they should do so in the interval between θ_1 and θ_2; the perfection of this cancelling being indicated by the closeness of the agreement between the means of the four damping coefficients obtained from $\theta_1 \theta_2$ and the corresponding mean from $k_1 k_2$.

This agreement thus comes to be a measure of the freedom of the mean value of θ_1 from error due to pendulum swings, and it was for this reason as well as for its bearing on the theory of the R effect (see above) that the following comparison of the two sets of damping coefficients was of interest.

In Table VIII. are collected the data for this comparison. The numbers given are the percentage differences between the means of each group of four damping coefficients (a, b, c, d) calculated from $\theta_1 \theta_2$ or $\theta_2 \theta_3$ and from $k_1 k_2$ respectively, the sign + meaning

TABLE VIII.

	Specimen 10.		Specimen 11.	
	$\theta_1\theta_2$	$\theta_2\theta_3$	$\theta_1\theta_2$	$\theta_2\theta_3$
A	−1·2	+1·1	+1·3	+1·9
	−0·3	+2·2*	+2·1	−1·8
	+0·6		−0·7	+4·2*
	+1·6			
	+1·3		−1·6	+1·6
	−2·3		+1·6	+1·7
	+1·0		+1·1	+5·0*
	0·0			
			+2·0	
B	+0·8		+4·8 (?)	
	−0·2			
	−0·6		+1·7	
	+0·5		−0·4	
C	+0·1			
	0·0			
D	+0·4	−2·8		
	+0·2	+1·0		
Means	+0·12	−0·2	+0·79	+0·8

that the θ value is the greater. The horizontal lines indicate a change in the suspension, and, in the case of specimen 10, the data are for the same determinations as are recorded in Tables III. and IV., the letters A, B, C, D having the same meaning in both cases.

Specimen 11, the only other for which θ_2 and θ_3 were recorded, was for some reason extremely difficult to adjust in the helix, and it was not until several determinations of the Ratio had been made that we found the position of the helix which brought the pendulum swings within bounds.

The Ratios obtained from these earlier determinations were all abnormally high, and were discarded when the correct adjustment was hit upon ; and it is to these earlier ones that the data for Specimen 11 in Table VIII. belong.

The Table contains all the data we have, but in calculating the means we have allowed ourselves certain liberties for the following reasons :—Against the 4·8 per cent. of Specimen 11, which is followed by a query, there is in our record a note to the effect that some special disturbing cause was at work when this was taken, which rendered the zeros uncertain. As it is so much larger than any of the others it seems reasonable to rule it out.

Three of the θ_2 θ_3 percentages are marked with an asterisk. In these cases θ_3 was so small owing to rapid damping (from 0·5 to 0·75 mm.) that the values of θ_2 θ_3 were too doubtful to rely upon, and we omitted them also.

The means in the last line were taken, with these four omissions ; and as each value of θ represents about 40 observations, the means for $\theta_1 \theta_2$ depend on about 1000 pairs of observations, and those for θ_2 θ_3 on about 300.

Whether the values for Specimen 11 are higher than for Specimen 10 owing to the less perfect adjustment of the former it is impossible to say, but it may well be so. In any case it appears that the mean difference between the specimens is only a matter of a few tenths of 1 per cent.

In our view this implies that in the interval of time between θ_1 and θ_2 the effects of pendulum swing cancel one another when the means of the four cases, a, b, c, d, are considered ; and as the same cancelling process is to be expected during the time required for the first throw, θ_1, we conclude that the mean of the four values of θ_1 is for practical purposes free from error due to pendulum swing.

Lastly, as to the interpretation of the θ_2 θ_3 dampings in Table VIII. The position is that if they are of opposite sign to those from θ_1 θ_2 they afford evidence of the gradual appearance of momentum in the iron for many seconds after the magnetic reversal ; while if of the same sign they make it probable that the true damping in the case of the R effect really does slightly exceed that obtained from k.

The data are obviously insufficient for deciding between these alternatives with certainty. If we take means of all the dependable figures—and it is the best that can be done—the value for θ_1 θ_2 is $+0·36$ and for θ_2 θ_3 is $+0·4$, which points to an excess of about one-third per cent. of the coefficients for θ over those for k.

There is nothing unlikely in this as the k deflections were of the order of a couple of centimetres, while θ was only a few millimetres ; and it is, moreover, borne out by a long series of measurements we made on the damping of Specimen 10 when in condition A (Table III.).

The means of these are given in Table IX., where the first column contains numbers of which each is the mean of about a dozen values of k_1 in millimetres (in the case of the last about double that number), and the second the corresponding values of $1+\lambda$. The ratio k_1 k_2 in these measurements was about 3·7, and throws were taken in both directions.

TABLE IX.

k_1 (mm.).	$1+\lambda$.
49·5	1·628
45·3	1·632
38·0	1·637
30·7	1·633
20·9	1·632
13·4	1·635
7·2	1·639

The last value of $1+\lambda$ and the last but two correspond to throws of the same order of magnitude as those for θ and k in the actual experiments on which Table VIII. is based ; and it will be seen that they differ by an amount which is in the right direction and of the right order of magnitude (0·4 per cent.) to account for the values 0·36 per cent. and 0·4 per cent. obtained from Table VIII. The agreement is indeed so close that this can only be the result of accident for such rough data, but it none the less supports the view that the damping coefficients for the small deflections which occurred when the R effect was measured were probably a few tenths per cent. higher than the coefficients obtained from k. 0·002 has therefore been added to the measured values of the Ratio before entering them in Table V.

Eddy Currents.

In the work both of STEWART and of EINSTEIN and DE HAAS experiments were made to determine the effect, if any, of eddy currents in the suspended system upon the value of the Ratio ; STEWART suspending a silver wire, and EINSTEIN and DE HAAS a copper wire, in place of the magnetic specimen. In each case it was concluded that any effect present was inappreciable.

The eddy currents in copper and silver are, however, small compared with those in iron ; for the effect of the low permeability of these two metals in reducing the currents far outweighs that of their higher conductivity in increasing them.

Even in the case of the silver wire where the disadvantage was lessened by the use of a very strong magnetic field, STEWART was only able to say that the effect, if any, of the eddy currents was less than 40 per cent. of the corresponding R effect; the throw he actually obtained with the silver being "certainly less than 0·2 mm.," whereas it should have been 0·5 mm. if an R effect corresponding to the magnetic conditions had been present.

It therefore seemed desirable to make sure that the apparent absence of any tendency in the copper or the silver to rotate on reversal of the field was not due to the difficulty of detecting it. For this purpose a specimen, cut from the same coil as No. 10 (diameter, 0·36 mm.), was prepared by coating it electrolytically with a sheath of copper about 0·12 mm. thick (deposited by 3 m.amps. flowing for 48 hours). We were thus able to combine the advantages of the conductivity of copper and the permeability of iron with the result that, instead of being reduced, the average magnetic moment of the eddy currents, for a given rate of change of the magnetic induction, was increased about 25 times, and their angular momentum about 14 times as compared with the corresponding values for the iron core alone.

As STEWART points out, the E.M.F. which sets up the eddy currents, imparts an impulse to the positive electricity present, which is in the same direction as the spin of the electron gyrostats responsible for the magnetism of the specimen; and since there is as much negative as positive electricity in the metal, it imparts an equal and opposite impulse to the negative electricity present.

The only chance of any measurable throw arising from these two opposing impulses lies in the fact that they are not imparted to the rigid structure of the metal simultaneously. While that received by the positive electricity is given up to the metal at once, the negative electricity retains the effect of its impulse as motion of the electrons until, in coming to rest, they too impart their impulse to the metal, and the eddy currents die down.

Two conditions are necessary for obtaining a measurable effect—the opposing impulses must be large enough, and the time that elapses between their respective arrivals at the metal must be appreciable compared with the quarter period of the suspended system—and, as appears from what follows, neither condition is fulfilled, even in the extreme case of the copper sheathed iron.

Since the magnetised specimen is very small the helix field, H_1, and the magnetising current, C, are proportional. The growth of the field is therefore calculable from the resistance (13 ohms) inductance (0·015 henry), and E.M.F. (40 volts) in the helix circuit, and the turns per centimetre of the helix (33) by the equations

$$t = \frac{0 \cdot 015}{13} \log_e \frac{C_0}{C_0 - C}, \quad \text{and} \quad H_1 = 4\pi \times 33 \times C,$$

where t is the time in seconds reckoned from the closing of the helix circuit.

The eddy currents produce a field H_2 in the iron, which is of opposite sign to, and has to be subtracted from, H_1 to obtain the true magnetising field, H. If B is the induction at any moment in the iron, c the eddy current in the copper sheath, R the resistance of the sheath to this current ($2 \cdot 2 \times 10^{-5}$ ohm), a the cross-section of the iron ($1 \cdot 02 \times 10^{-3}$ sq. cm.) and l the length of the sheath (10 cm.)

$$c = \frac{a\, d\mathrm{B}/dt}{\mathrm{R}}, \quad \mathrm{H}_2 = 4\pi \mathrm{C}/l, \quad \text{and} \quad \mu(\mathrm{H}_1 - \mathrm{H}_2) = \mu \mathrm{H} = \mathrm{B},$$

where the permeability μ is obtained as follows :—

When the magnetism in the specimen is reversed half the hysteresis loop of the B–H curve is traversed from the point where it cuts the + axis of B (at H = 0) to the corresponding point on the − axis, or *vice versa*. It is from one of these points regarded as the origin that we have reckoned values of B and H, the ratios of which are represented by μ in the above equation.

By the use of a rough geometrical method and neglecting the possibility of time lag between the application of the magnetising field and the appearance of the induction in the iron, approximate values of B have been obtained in terms of H_1, and therefore of t, which are given in Table X.

TABLE X.

H_1.	t.	μ.	B.	$d\mathrm{B}/dt$.
0	$0 \cdot 0 \times 10^{-4}$	1100	0	
1	$0 \cdot 97 \times 10^{-4}$	1590	660	$0 \cdot 68 \times 10^{7}$
2	$2 \cdot 05 \times 10^{-4}$	2040	2000	$1 \cdot 24 \times 10^{7}$
3	$3 \cdot 19 \times 10^{-4}$	2380	3980	$1 \cdot 74 \times 10^{7}$
4	$4 \cdot 51 \times 10^{-4}$	2460	6640	$2 \cdot 01 \times 10^{7}$
5	$5 \cdot 91 \times 10^{-4}$	2410	9380	$1 \cdot 96 \times 10^{7}$
6	$7 \cdot 60 \times 10^{-4}$	2260	12,150	$1 \cdot 64 \times 10^{7}$
8	$11 \cdot 77 \times 10^{-4}$	2000	16,610	$1 \cdot 07 \times 10^{7}$
10	$18 \cdot 4 \times 10^{-4}$	1800	19,500	$0 \cdot 43 \times 10^{7}$

The maximum value of the eddy current is thus

$$\mathrm{C}_{\max} = \frac{1 \cdot 02 \times 10^{-3}}{2 \cdot 2 \times 10^{-5} \times 10^{9}} (d\mathrm{B}/dt)_{\max} = 0 \cdot 93 \text{ abs.}$$

The momentum in unit length of the path of the electrons is equal to the mass they carry per second past any cross section of their path, and this again is equal to $\mathrm{C}m/e$. Hence, if r is the mean radius of the sheath (assumed thin) the angular momentum of the electrons in it is

$$2\pi r^2 \mathrm{C}m/e,$$

which for the maximum value of C just obtained is 2×10^{-10}; roughly 2000 times less than that of the R effect, and of course still less when the sheath is absent.

The further fact which may be deduced from the last column of the table, that the eddy currents have practically died down in a few thousandths of a second, is thus superfluous to prove that their momentum is entirely too small to influence the measurements of the R effect. There was, however, just the chance that some unsuspected direct action of the field on the eddy currents might result in an incompletely eliminated throw error, and for this reason we persevered in the experiments with the copper sheath.

Table XI. contains the results of the five determinations made. The higher moment of inertia in those for large damping is due to an added vane, and classes them with the data for curve B; the three first Ratios corresponding with A.

TABLE XI.

Ratio.	$1+\lambda$.	I.	Ratio from curves.	Percentage difference.
0·498	1·11	$2 \cdot 84 \times 10^{-4}$		
0·492	1·11	$2 \cdot 84 \times 10^{-4}$		
0·493	1·11	$2 \cdot 84 \times 10^{-4}$		
Mean 0·494	—	—	0·509 (A)	−3·0
0·538	1·905	$7 \cdot 43 \times 10^{-4}$		
0·522	1·903	$7 \cdot 95 \times 10^{-4}$		
Mean 0·530	—	—	0·528 (B)	+0·4
Weighted mean				−1·6

The last column shows the percentage difference between the measured Ratio and that from the curve at the same damping, the mean difference being 1·6 per cent.; and as the presence of the sheath magnifies the eddy current many times, it is safe to conclude that, in the ordinary unsheathed specimens we used, an eddy current effect on the value of the Ratio must have been at most a small fraction of 1 per cent.

SUMMARY.

RICHARDSON has shown that the angular momentum arising in a ferro-magnetic substance from unit change in its magnetic moment should have the value $1 \cdot 13 \times 10^{-7}$ if gyrating electrons are responsible for its magnetism. Measurements of this quantity by the ballistic method for three specimens of iron and one of nickel are given in the paper. The results, divided by $1 \cdot 13 \times 10^{-7}$, are collected in the following table under "Ratio." They agree to within $1\frac{1}{2}$ per cent. with one another, and their mean is 0·6

per cent. greater than 0·500. Close proportionality is also shown to exist between the change of magnetic moment and the angular momentum resulting.

Specimen.	Ratio.
Iron	0·507
,, ...	0·501
,, ...	0·500
Nickel	0·505

Errors of measurement are discussed in detail, and, as far as can be judged, shown to be eliminated.

The specimen experimented upon consisted of an upright wire suspended by a quartz fibre. By the introduction of a hinged joint between the wire and the fibre, the important adjustment of the magnetic axis of the wire to the vertical is so much facilitated that measurements were found to be practicable on reversal of the magnetism instead of on merely reducing it to zero. The more perfect symmetry resulting from this procedure may be the cause of the more consistent results obtained.

The effect on the Ratio of the eddy currents set up in the specimen by its changing magnetism was examined and shown to be not more than a small fraction of 1 per cent. for the specimens used.

At high dampings the ordinary damping correction was found to give values that were too large; the error being more noticeable for small than for large moments of inertia. The effect of this on the measured Ratio is shown to be eliminated.

The cost of the present work has been borne by the University of Bristol Colston Research Society, and we have pleasure in gratefully acknowledging our indebtedness to that body.

To Prof. TYNDALL our thanks are especially due, not only for the generous way in which he has constantly facilitated the work, but also for the interest he has taken in it from the beginning and for the many helpful discussions we have had with him during its progress.

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